Ermüdungsanalyse von anistropen Kupfer-Durchkontaktierungen in der Leiterplatte
Fatigue analysis of anisotropic copper-vias in a circuit board

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Kurzfassung

Die Zuverlässigkeit der Leiterplatte unter thermischer Belastung ist gemäß der Ermüdung der plastisch deformierenden Kupfer-Durchkontaktierungen bestimmt. Die Durchkontaktierungen aus Kupfer sind elektrolytisch abgeschieden. Dadurch weist das Material kubische Symmetrie im Einkristall aus – Kupfer ist anisotrop. Die Korngrößen sind in Bezug auf die geometrischen Dimensionen genügend klein. Daher wird vermutet, dass die Durchkontaktierung ein isotropes Verhalten aufweist, womit eine Analyse mittels der finiten Elementmethode durch Benutzung von kommerziellen Programmen möglich ist. Eine numerische Berechnung von plastischen Deformationen der anisotropen Materialien mittels der finiten Elementmethode ist jedoch nur unter Anwendung von Forschungscodes realisierbar. Zuerst fassen wir die Plastizitätstheorie für Metalle zusammen, dann wenden wir diese auf die Simulation einer Leiterplatte unter Anwendung von quelloffenen Programmpaketen an. Diese Berechnung ermöglicht eine Ermüdungsanalyse für die Kupfer-Durchkontaktierung und die Lebensdauer-Abschätzung der Leiterplatte.

Abstract

The reliability of a circuit board under thermal loading is restricted by fatigue of plastically deforming copper-vias. Vias consist of electro-deposited copper, which possesses a cubic symmetry as a single crystal—in other words the copper is anisotropic. The grain size of electrodeposited copper is small compared to the geometric size of the via. Hence it is usually assumed that the via behaves isotropically in order to enable a finite element analysis of plasticity with commercial softwares. However, a finite element analysis of anisotropic materials deforming plastically is possible by using research codes. Therefore, we first briefly outline the theory of plasticity for metallic compounds and, second, apply it to a multi-layered circuit board by using open-source packages. This computation allows us to perform a fatigue analysis for copper vias in the circuit board and to predict their lifetime.

1 Variational formulation for associated plasticity

The plastic deformation of an engineering structure is of interest. We model the structure as a *continuum body*, $\mathcal{B}$, in the *reference* configuration. The reference configuration is taken as the *initial* configuration in solid body mechanics. The deformation has to satisfy the balance of linear momentum in the initial configuration:

$$\rho_0 \frac{\partial^2 u_i}{\partial t^2} - \frac{\partial P_{ji}}{\partial X_j} - \rho_0 f_i = 0 \,, \tag{1}$$

where the initial mass density, ρ_0, and the body forces per mass, f_i, are known. The displacement, u_i, and the first PIOLA-KIRCHHOFF stress tensor are the unknowns. By relating the stress tensor to the displacements or its rates we will obtain the field equations describing the deformation of the structure. The deformation is characterized by the displacements, $u_i = x_i - X_i$, where x_i denote the positions of particles in the current (present) time and X_i in the reference (initial) time. We exclude any geometric nonlinearities by assuming that the deformation gradient can be approximated as follows:

$$F_{ij} = \frac{\partial x_i}{\partial X_j} = \delta_{ij} + \frac{\partial u_i}{\partial X_j} \approx \delta_{ij} \,, \tag{2}$$

which is a consequence of the fact that the displacement gradients (thus strains) are smaller than the geometric dimensions. This assumption results in the so-called CAUCHY stress tensor, σ_{ij}, being identical to the first PIOLA-KIRCHHOFF stress tensor. Therefore, the balance of momentum reads

$$\rho_0 \frac{\partial^2 u_i}{\partial t^2} - \frac{\partial \sigma_{ji}}{\partial X_j} - \rho_0 f_i = 0 \,. \tag{3}$$

The CAUCHY stress tensor is symmetric, $\sigma_{ij} = \sigma_{ji}$. Moreover, neglecting the geometrical nonlinearities leads to the

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linear strain definition:

$$\varepsilon_{ij} = \frac{1}{2}\Big(\frac{\partial u_i}{\partial X_j} + \frac{\partial u_j}{\partial X_i}\Big) = \frac{\partial u_{(i}}{\partial X_{j)}} \ . \tag{4}$$

Hence the strain tensor is also symmetric, $\varepsilon_{ij} = \varepsilon_{ji}$. Since we restrict ourselves to linear strains, their additive decomposition is admissible:

$$\varepsilon_{ij} = {}^{\mathrm{e}}\varepsilon_{ij} + {}^{\mathrm{t}}\varepsilon_{ij} + {}^{\mathrm{p}}\varepsilon_{ij} \ , \tag{5}$$

where elastic, thermal, and plastic strains are denoted by ${}^{\mathrm{e}}\varepsilon_{ij}$, ${}^{\mathrm{t}}\varepsilon_{ij}$, and ${}^{\mathrm{p}}\varepsilon_{ij}$, respectively. For the elastic part of strains we use HOOKE's law:

$$\sigma_{ij} = C_{ijkl}\,{}^{\mathrm{e}}\varepsilon_{kl} = C_{ijkl}\big(\varepsilon_{kl} - {}^{\mathrm{t}}\varepsilon_{ij} - {}^{\mathrm{p}}\varepsilon_{ij}\big) \ . \tag{6}$$

The latter is a linear relation between stress and elastic strain. Because of the linear relation and due to the symmetry of the stress and strain tensors the following identities for the stiffness tensor, C_{ijkl} hold

$$C_{ijkl} = C_{klij} \,,\ C_{ijkl} = C_{jikl} \,,\ C_{ijkl} = C_{ijlk} \ . \tag{7}$$

Therefore, the stiffness tensor possesses 21 parameters, in other words, 21 independent components must be measured. These parameters are easily identifiable when using VOIGT's matrix notation with I, J running from 1 to 6 such that:

$$C_{IJ} = \begin{pmatrix} C_{1111} & C_{1122} & C_{1133} & C_{1123} & C_{1113} & C_{1112} \\ & C_{2222} & C_{2233} & C_{2223} & C_{2213} & C_{2212} \\ & & C_{3333} & C_{3323} & C_{3313} & C_{3312} \\ & & & C_{2323} & C_{2313} & C_{2312} \\ & \text{sym.} & & & C_{1313} & C_{1312} \\ & & & & & C_{1212} \end{pmatrix} . \tag{8}$$

For the thermal strains we use the so-called DUHAMEL-NEUMANN law:

$${}^{\mathrm{t}}\varepsilon_{ij} = \alpha_{ij}(T - T_{\mathrm{ref.}}) \ , \tag{9}$$

where the reference temperature, $T_{\mathrm{ref.}}$, and the coefficients of thermal expansions (CTE), α_{ij}, are material specific parameters. In case of certain material symmetries the number of parameters decreases. For an orthotropic material, i.e., in the present case the laminate, 9 stiffness and 3 CTE parameters are necessary. The stiffness matrix (in MPa) and the CTE tensor in ppm/K, both at room temperature, can be obtained from experimental measurements [2], such that:

$$C^{\mathrm{L}}_{IJ} = \begin{pmatrix} 66242 & 41797 & 37814 & 0 & 0 & 0 \\ & 50460 & 32290 & 0 & 0 & 0 \\ & & 31591 & 0 & 0 & 0 \\ & & & 2250 & 0 & 0 \\ & \text{sym.} & & & 2250 & 0 \\ & & & & & 6630 \end{pmatrix} ,$$
$$\alpha^{\mathrm{L}}_{ij} = \begin{pmatrix} 13.2 & 0 & 0 \\ 0 & 16.7 & 0 \\ 0 & 0 & 39 \end{pmatrix} . \tag{10}$$

For a material belonging to the cubic class, as copper, the amount of parameters reduces to 4 and the stiffness matrix in GPa reads with the constants from [8, Table 10] and CTE in ppm/K (or equivalently in µm/(m K)) as in [4]:

$$C^{\mathrm{C}}_{IJ} = \begin{pmatrix} 169.1 & 122.2 & 122.2 & 0 & 0 & 0 \\ & 169.1 & 122.2 & 0 & 0 & 0 \\ & & 169.1 & 0 & 0 & 0 \\ & & & 75.42 & 0 & 0 \\ & \text{sym.} & & & 75.42 & 0 \\ & & & & & 75.42 \end{pmatrix} ,$$
$$\alpha^{\mathrm{C}}_{ij} = \begin{pmatrix} 17 & 0 & 0 \\ 0 & 17 & 0 \\ 0 & 0 & 17 \end{pmatrix} . \tag{11}$$

In order to model plasticity we need to take the rate of plastic strain into account and obtain from Eq. (6):

$$\sigma^{\bullet}_{ij} = C_{ijkl}\big(\varepsilon^{\bullet}_{kl} - {}^{\mathrm{t}}\varepsilon^{\bullet}_{kl} - {}^{\mathrm{p}}\varepsilon^{\bullet}_{kl}\big) \ . \tag{12}$$

The rate of plastic strain is modeled using *associated plasticity* with a so-called *flow potential*, f, and a positive multiplier, $\Lambda^{\bullet}$, as follows:

$${}^{\mathrm{p}}\varepsilon^{\bullet}_{ij} = \Lambda^{\bullet}\frac{\partial f}{\partial \sigma_{ij}} \ . \tag{13}$$

We will simulate cyclic loading. Hence a kinematic hardening law is implemented. The plasticity is associated with the deviatoric part of the stress tensor:

$$\sigma_{|ij|} = \sigma_{ij} - \frac{1}{3}\sigma_{kk}\delta_{ij} \ , \tag{14}$$

and an equivalent yield stress, σ_{Y}, being a material specific parameter. Kinematic hardening is modeled by using a flow potential proposed by [11] in this form:

$$f = \frac{1}{2}\big(\sigma_{|ij|} - \beta_{ij}\big)\big(\sigma_{|ij|} - \beta_{ij}\big) - \frac{1}{3}\sigma^2_{\mathrm{Y}} \ , \tag{15}$$

where the so-called *back stress*, β_{ij}, needs to be modeled next. The flow potential vanishes for the case of plasticity, $f = 0$, and it remains zero as long as plastic deformation occurs, $f^{\bullet} = 0$. Now, by using these conditions and the model in [12] we obtain an equation for the unknown back stress after some calculations:

$$\beta^{\bullet}_{ij} = \frac{(\sigma_{|kl|} - \beta_{kl})\sigma^{\bullet}_{kl}}{\frac{2}{3}\sigma^2_{\mathrm{Y}}}\big(\sigma_{|ij|} - \beta_{ij}\big) \ . \tag{16}$$

Moreover, by using Eq. (13) and the yielding conditions we obtain

$$\begin{aligned} \frac{\partial f}{\partial \sigma_{ij}} &= \sigma_{|ij|} - \beta_{ij} \ , \\ \Lambda^{\bullet} &= \frac{(\sigma_{|ij|} - \beta_{ij})C_{ijkl}\varepsilon^{\bullet}_{kl}}{\frac{4}{9}h\sigma^2_{\mathrm{Y}} + (\sigma_{|ij|} - \beta_{ij})C_{ijkl}(\sigma_{|kl|} - \beta_{kl})} \ . \end{aligned} \tag{17}$$

In order to close Eq. (3) we need to obtain σ_{ij}. However, by using Eqs. (12), (9) we can find the rate of stress. For a numerical solution this can be incrementally handled by using the following time discretization according to the *finite difference method*:

$$\dot{\sigma}_{ij} = \frac{\sigma_{ij} - \sigma^0_{ij}}{\Delta t} \, , \; \sigma_{ij} = \sigma^0_{ij} + \Delta t \dot{\sigma}_{ij} \, , \tag{18}$$

where the upper index "0" denotes to the (known) value of stress in the last time step. Therefore, the variational form acquired for Eq. (3) after time discretization reads

$$\mathrm{F} = \int_{\mathcal{B}} \left(\rho_0 \frac{u_i - 2u^0_i + u^{00}_i}{\Delta t \Delta t} \delta u_i + \sigma_{ji} \frac{\partial \delta u_i}{\partial X_j} - \rho_0 f_i \delta u_i \right) \mathrm{d}V - \\ - \int_{\partial \mathcal{B}^{\mathrm{N}}} \hat{t}_i \delta u_i \, \mathrm{d}A \, , \tag{19}$$

where the traction vector $\hat{t}_i$ is given on the boundary, $\partial\mathcal{B}$. The test functions, δu_i, shall be from the same space as the unknowns, u_i, i.e., we use the GALERKIN approach in the *finite element method*. In each time increment, the stress rate in Eq. (12) and the back stress are approximated by using the values from the last time step such that:

$${}^{\mathrm{p}}\dot{\varepsilon}_{mn} = \langle \gamma \rangle \frac{(\sigma^0_{|ij|} - \beta^0_{ij}) C_{ijkl} \dot{\varepsilon}_{kl} (\sigma^0_{|mn|} - \beta^0_{mn})}{\frac{4}{9} h \sigma^2_{\mathrm{Y}} + (\sigma^0_{|ij|} - \beta^0_{ij}) C_{ijkl} (\sigma^0_{|kl|} - \beta^0_{kl})} \, , \\ \beta_{ij} = \beta^0_{ij} + \Delta t \dot{\beta}_{ij} \, , \; \dot{\beta}_{ij} = \frac{(\sigma^0_{|kl|} - \beta^0_{kl}) \dot{\sigma}_{kl}}{\frac{2}{3} \sigma^2_{\mathrm{Y}}} (\sigma^0_{|ij|} - \beta^0_{ij}) \, , \tag{20}$$

The parameter γ in HEAVISIDE brackets, $\langle\gamma\rangle$, enables us to distinguish between plastic or elastic deformation by performing a Boolean query over coordinates. It has the value 0 for elastic and the value 1 for plastic regions of $\mathcal{B}$. We compiled all materials data at room temperature, 20 °C, for Cu-ETP (3N purity) taken from [4], [13] and for laminate taken from [7] in Table 1.

Table 1 Materials data of copper and laminate.

	ρ in 10^3 kg/mm^3	σ_{Y} in MPa	h in MPa
copper	$8.94 \cdot 10^{-9}$	100	615
laminate	$2.5 \cdot 10^{-9}$	-	-

By implementing sufficiently small time steps, this approximation converges to the real solution. Such a variational form can only be solved by using research codes. We apply the open-source codes developed by the FEniCS project, see [5], [9], [3].

2 Modeling a multi-layered circuit board and its lifetime prediction

We model a standard multilayer circuit board composed of an FR4 composite board and copper vias. We assume that the room temperature, 20 °C, is the reference temperature, $T_{\mathrm{ref.}}$, where no thermal stresses occur. By altering the temperature between −40 °C and 125 °C the circuit board undergoes a plastic deformation within the copper vias. The accumulated plastic strain is calculated by using the VON MISES equivalent strain:

$${}^{\mathrm{p}}\varepsilon_{\mathrm{acc.}}(t) = \int_0^t \sqrt{\frac{2}{3} {}^{\mathrm{p}}\dot{\varepsilon}_{ij} {}^{\mathrm{p}}\dot{\varepsilon}_{ij}} \, \mathrm{d}t \, . \tag{21}$$

The mesh and the distribution of the accumulated strain after at the end of a cycle are depicted in Fig. 1. A mean value

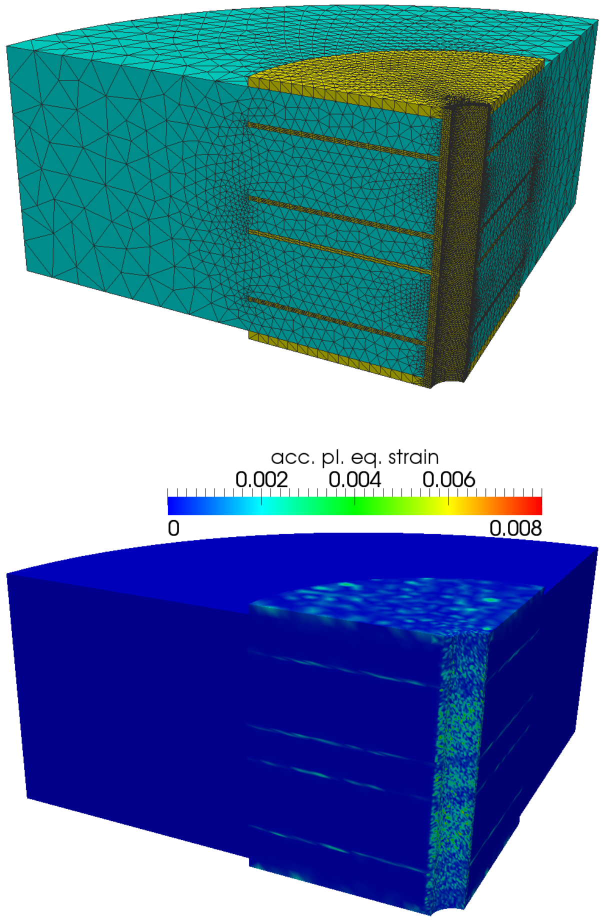

Figure 1 Mesh of the model, laminate in light green and copper in yellow (left); accumulated plastic strain at the end of a cycle (right inset).

for the accumulated plastic strain is obtained by averaging only over via, $\mathcal{B}_{\mathrm{via}}$, such that:

$${}^{\mathrm{p}}\bar{\varepsilon}_{\mathrm{acc.}}(t) = \frac{\int_{\mathcal{B}_{\mathrm{via}}} {}^{\mathrm{p}}\varepsilon_{\mathrm{acc.}}(t) \, \mathrm{d}V}{\int_{\mathcal{B}_{\mathrm{via}}} \mathrm{d}V} \, . \tag{22}$$

The change of this mean value after a cycle t_{end} represents a measure for fatigue and reads

$$\Delta {}^{\mathrm{p}}\bar{\varepsilon}_{\mathrm{acc.}} = {}^{\mathrm{p}}\bar{\varepsilon}_{\mathrm{acc.}}(t_{\mathrm{end}}) - {}^{\mathrm{p}}\bar{\varepsilon}_{\mathrm{acc.}}(t=0) \, . \tag{23}$$

For metals subject to cyclic thermal loading the following relation holds, see [10]:

$$\Delta {}^{\mathrm{p}}\bar{\varepsilon}_{\mathrm{acc.}} = D^{0.6} (N_{\mathrm{f}})^{-0.6} \, , \; D = \ln \left(\frac{100}{100 - R} \right) \, . \tag{24}$$

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The reduction of the cross section in breakage, R, for electro-deposited copper is set to 60, which indicates 60 % change of cross sectional area by tensile test. We can simulate and predict the lifetime of vias made of copper with cubic symmetry in any orientation. In order to investigate the role of the orientation, we perform nine simulations. In each simulation the copper material of cubic symmetry has been oriented by a random distribution such that the predicted lifetime differs in each simulation. The results are compiled in Table 2.

Table 2 Accumulated plastic strain in a cycle and its corresponding lifetime for different simulations with randomly oriented copper material of cubic symmetry.

	$\Delta^{\mathrm{p}}\bar{\varepsilon}_{\mathrm{acc.}}$ in 10^{-5}	N_{f}
Sim. I	101.39	67 736
Sim. II	101.16	68 000
Sim. III	101.98	67 085
Sim. IV	101.30	67 837
Sim. V	102.26	66 783
Sim. VI	102.33	66 701
Sim. VII	101.74	67 353
Sim. VIII	101.40	67 723
Sim. IX	102.54	66 477

3 Conclusion

Thermomechanical fatigue for a multilayer electric board has been implemented by using research codes. In this way we have easily succeeded modeling a randomly distributed cubic material for electro-deposited copper used for vias. Due to the small size of the grains copper is usually modeled as an isotropic material. We have analyzed this assumption by comparing the predicted lifetime in nine simulations. In each simulation the orientation has been assigned randomly, such that the predicted lifetime differs in each simulation. However, there is no significant change in the lifetime prediction since the accuracy of such a prediction is many times smaller than the differences between the predicted lifetimes in each simulation. We publish the code used for computation in [1] under [6] in order to encourage scientific studies and have left a more detailed inspection for determining the best orientation of copper vias (regarding lifetime) to further research.

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